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Nonlinear Dynamics in Economics

A Theoretical and Statistical Approach to Agricultural Markets

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Contents

Chapter 1

Introduction

1.1 Introduction

In economics, one often observes time series that exhibit different patterns of qualitative behavior, both regular and irregular, symmetric and asymmetric. There exist two different perspectives to explain this kind of behavior within the framework of a dynamical model. The traditional belief is that the time evolution of the series can be explained by a linear dynamic model that is *exogenously* disturbed by a stochastic process. In that case, the observed irregular behavior is explained by the influence of external random shocks which do not necessarily have an economic reason. A more recent theory has evolved in economics that attributes the patterns of change in economic time series to an underlying nonlinear structure, which means that fluctuations can as well be caused ***endogenously*** by the influence of market forces, preference relations, or technological progress. One of the main reasons why nonlinear dynamic models are so interesting to economists is that they are able to produce a great variety of possible dynamic outcomes — from regular predictable behavior to the most complex irregular behavior — rich enough to meet the economists' objectives of modeling.

The traditional linear models can only capture a limited number of possible dynamic phenomena, which are basically convergence to an equilibrium point, steady oscillations, and unbounded divergence. In any case, for a linear system one can write down exactly the solutions to a set of differential or difference equations and classify them. Explosive behavior or divergence implies a breakdown of the market. In general, such behavior is ruled out because it is not empirically observed. Sustained oscillations or cycles represent the most complicated behavior one can think of in a linear model.

However, if one assumes that the observed time series pertains to a specific cycle, such as the business cycle, and the underlying dynamic structure is presumed to be linear, then the cycle has to be of a symmetric nature. Linear modeling of the business cycle hence implies that the upswing of the cycle must be symmetric to its downswing, as well as the peaks are symmetric to the troughs. In addition, a permanent oscillation of the variables is only likely to occur for certain parameter values, and hence can be considered as a rare phenomenom. Most often, the aim of linear dynamic modeling in economic theory is to characterize the necessary and sufficient conditions for the stability of the equilibrium.

Nonlinear dynamic models go beyond the scope of a linear model since they can capture both regular and irregular behavior, asymmetric cycles, and even erratic behavior. They can give rise to completely aperiodic time paths that can deceive every statistician. Without prior knowledge of the system of equations that generated the time path, it can hardly be distinguished from a random sequence and will pass the standard statisticians' tests of randomness. A chaotic time path can be subject to quasi cyclical behavior, or display sharp breaks in the qualitative pattern. In a chaotic regime no pattern ever repeats itself – otherwise it would be periodic. Even if the system is precisely known, long-term prediction is impossible. The reason is that closely neighboring points – assume one point to be the true state of the system and the other point to be the measurement point, which is naturally subject to a round-off error, be it miniscule – can produce completely different time paths after a few iterations. This astonishing property of nonlinear deterministic systems is responsible for the name "chaos"[1] and, under the term "sensitivity to initial conditions", has become part of the definition of chaos.

In the behavioral sciences, such as economics, a nonlinear approach enables us to model seemingly random behavior as the outcome of the agents' optimisation problem where it can pay-off to the agent to purposely behave in a stochastic manner.

In economics, the possibility of chaotic behavior has been shown in models of growth cycles (Day [1983], Dana and Malgrange [1984]), in cobweb models (Jensen and Urban [1982], Chiarella [1988], Hommes [1991]), in duopoly models (Rand [1978]), or in models with interdependent consumer preferences

[1] Unfortunately, this expression is completely misleading. The term "chaos" is normally used to refer to the absence of any structures or rules, but here in fact, the opposite is the case. In comparison to the limited behavior of linear dynamic models, one might simply call the aperiodic time evolution of a nonlinear model "chaotic", whereas "interesting" would fit better.

(Gaertner [1987], Gaertner and Jungeilges [1988]). Overlapping generations models are of special interest because they illustrate that chaotic dynamics can be the outcome of an intertemporal utility maximisation program (Benhabib and Day [1981], Grandmont [1985]). This is just to mention a few authors of an immense body of literature on chaos in economics. For surveys of chaotic behavior in economic models, the interested reader is referred to Lorenz [1989], Kelsey [1988], Baumol and Benhabib [1989], and Anderson, Arrow, and Pines [1988].

In recent years, techniques that can, in principle, distinguish between chaotic and stochastic data have been developed mainly in the natural sciences (see Eckmann and Ruelle [1980] for a review). Such methods have been applied to economic data by Barnett and Chen [1988], who could not deny the presence of chaos in monetary aggregates. Brock and Sayers [1988] asked the question whether the business cycle is characterized by deterministic chaos and concluded that the evidence of low-dimensional chaos is weak in the data examined. Frank and Stengos [1988] investigated Canadian income accounts for evidence of chaos and found that deterministic chaos is not compatible with the data. Frank, Stengos, and Gencay [1988] also looked at the gross national products of Japan and some European countries and came to the conclusion that none of the time series seem to be chaotic. Considering the financial data sets, Scheinkman and LeBaron [1986] used a shuffling data test and claim to have detected chaos in stock prices. We will return to some of the techniques applied by these authors in chapter 3.

In general, the detection of nonlinear structures in time series is a question of the quality and quantity of the data at hand. First of all, an abundance of data is required in order to get unequivocal results. Most economic data sets are rather short in comparison to data sets from the physical sciences. Secondly, most existing empirical investigations show that there is a better chance to detect nonlinear patterns in disaggregated series that is also measured on a small scale. For instance, by studying time series from biological systems, Sugihara, Grenfell and May [1990] concluded that on a city-by-city scale, measles exhibits low-dimensional chaos, whereas on a country-wide scale the behavior appears as a stochastically disturbed cycle. Furthermore, it is important that the data is collected in relatively small time intervals, say on a daily or weekly basis when considering economic time series. It seems to be more difficult to detect nonlinear patterns in aggregated macroeconomic time series, such as income accounts, than in microeconomic time series, such as financial data.

This study is motivated by the "nonlinear" perspective. It consists of five chapters that have been written (and can be read) almost independently of each other. The first chapter gives an introduction to nonlinear dynamics to a reader who is not familiar with this topic. There are a lot of interesting features about chaos that attract both specialists and nonspecialists. The introductory chapter is restricted to a selection of properties, that are important to understand the remaining chapters. The fundamental ideas are stated in a somewhat informal style using simple, worked-out cases from the natural sciences. Attention is given to the one-dimensional quadratic map as it exhibits almost all interesting properties. Aside from its interesting theoretical properties, the nonlinear approach sheds a different light upon the traditional linear statistical methodology, which is outlined in view of the statistical part of the thesis.

The second chapter deals with one of the simplest dynamic models in economics, namely the cobweb model, which is often used to describe a highly stylized version of an agricultural market. It will be shown that the introduction of a nonlinear demand function to the cobweb model under adaptive expectations broadens the possible range of dynamic outcomes significantly. Although it can hardly be assumed that such a simplified model – be it linear or nonlinear – is able to perfectly explain the real world data, it is necessary to examine some observed time series from agricultural markets for typical nonlinear behavior such as the occurence of asymmetric cycles, low-dimensional clustering in phase space, and possible short-term predictability.

The third and the fourth chapters are concerned with detecting such special nonlinear patterns in the agricultural price series. The third chapter draws upon a selection of relatively new tests for (non)linearity, as suggested in the nonlinear time series literature. The fourth chapter deals with a near neighbor forecasting algorithm to distinguish chaos from measurement noise as recently proposed by Sugihara and May [1990]. This method is quite promising because it can be applied to relatively short data sets such as the economic time series. An extremely simple robust test will be proposed to accompany this algorithm in order to test its forecast performance, in case the data is noise infected.

The aim of the thesis is to show how complicated and interesting even the simplest dynamical system may become once the linearity assumption is given up. But, is it really worth investigating so much effort in nonlinear modeling? Looking at the data at hand, we can affirm that nonlinearities in agricultural time series cannot be considered as a rare phenomenom. Still, questions with respect to the modeling of such series are open and shall be outlined in chapter five.

1.2 The dynamics of the first order difference equations

In this section, a brief survey of the main properties of one-dimensional maps is given. Consider a first order difference equation as the simplest kind of a dynamical system

$$x_{t+1} = f(x_t). \tag{1.1}$$

For some initial value x_0, we want to know the evolution of the iterative process for $t \to \infty$

$$\{x_0, x_1, x_2, ...\} = \{x_0, f(x_0), f^2(x_0), ...\}$$

called *trajectory, discrete orbit or time path.* In a system with n degrees of freedom (n equals the number of first order difference or differential equations necessary to describe the system), an *attractor* is a subset of the n-dimensional state space, toward which almost all trajectories are attracted asymptotically. Once caught by the attractor, the trajectory will stay within or nearby the attractor forever. For a first order difference equation, the iterative process can, for example, converge to a fixed point or a periodic cycle. Yet, the attractor can also be of a more complicated nature, as we will see later. The set of initial states that are attracted toward the attractor is called the *basin of attraction.* The motion toward the attractor, or the sequence of iterations that have not yet reached the attractor, is called *transient.* We will say that we have a *periodic solution*, or a *cycle of period P*, if the iterative process is confined to a periodic orbit of finite period $P \geq 1$, such that P is the smallest natural number with $x = f^P(x)$. A stable *fixed point* of f is the special case of a period-one point i.e., $x = f(x)$, which is graphically given by the intersection point between the graph of f – also called the *phase curve* – and the 45^0 line.

If the function f in (1.1) is linear and hence monotonic, we can distinguish three types of dynamic behavior namely, exponential growth, convergence to a stationary point, and a cycle of period two, depending on the values of the slope and the intercept.

In comparison to a linear map, a simple nonmonotonic difference equation can give rise to strikingly complex behavior. Classical examples are the so-called *logistic or quadratic map*:

$$x_{t+1} = ax_t(1 - x_t) \quad \text{for} \quad a \in (0,4] \quad x_t \in [0,1], \tag{1.2}$$

and the continuous piecewise linear *tent map*,

$$x_{t+1} = \begin{cases} bx_t & \text{for} \quad 0 \leq x_t \leq \frac{1}{b} \\ \frac{b}{b-1}(1 - x_t) & \text{for} \quad \frac{1}{b} < x_t \leq 1 \end{cases} \qquad b > 1 \tag{1.3}$$

that has a discontinuous derivative at $x_t = \frac{1}{b}$.

We will now consider some properties of the so-called *unimodal maps* by illustrating them with the logistic map (1.2). Basic references to this subject are a classical treatment of this map by May [1976], and a monograph of Collet and Eckmann [1980]. The logistic map originates from population dynamics in a nonoverlapping generations framework, where x_t represents the relative population size of the tth generation and a stands for the growth rate.[2] However, it is exactly this map that also occurs in Day's [1982] economic growth model.

The logistic function specified in (1.2) maps the interval $I = [0,1]$ into itself (onto if $a = 4$), and belongs to the important class of *SN-unimodal functions* since it fulfills the following assumptions:

1. $f : I \to I$ is a continuous map on the unit interval $I = [0,1]$.
2. f is thrice differentiable and $f'(0) > 1$.
3. The function increases monotonically for $0 < x < x^*$, has a unique maximum at x^*, and decreases monotonically for $x > x^*$. For the logistic map the critical point is found at $x^* = \frac{1}{2}$.
4. The *Schwarzian derivative* is negative on $I - \{x^*\}$. We will call maps with negative Schwarzian derivatives *SN-maps* for short.[3]

For this class of maps, a neat theory has been worked out that establishes the unique existence of periodic solutions and their ordering:

Singer [1978] has shown that, for this class of maps, there exists *at most one stable periodic solution* towards which the critical point x^* is attracted.[4] If there is a stable periodic solution, then its basin of attraction has Lebesgue measure one and, therefore, almost all initial conditions are attracted by the periodic cycle.[5]

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[2] In order to ensure positive values of the relative population size x_t throughout the iterative process, the parameter a is restricted to the real interval [0,4]. A more generalized version of the quadratic map, where x and a are extended to the complex domain, can give rise to further interesting "strange attractors" such as the "San Marco attractor" which has been studied by Mandelbrot [1980].

[3] Confer appendix of chapter 2 for a concise treatment of the Schwarzian derivative.

[4] In fact, Singer's result is more general than it is stated here, since he shows that the number of periodic solutions, that are possibly stable, is related to the number of critical points.

[5] Note that the tent map is not an SN-map since it is not everywhere differentiable, and hence it may have more than one stable periodic solution.

A remarkable *theorem by Sarkovskii* (reported in Stefan [1977]) states that the periodic solutions are organized in the following ordering of natural numbers:

$$3 \succ 5 \succ 7 \succ \ldots$$

$$\succ 2^1 \times 3 \succ 2^1 \times 5 \succ 2^1 \times 7 \succ \ldots$$

$$\succ 2^k \times 3 \succ 2^k \times 5 \succ 2^k \times 7 \succ \ldots$$

$$\succ 2^k \succ \ldots \succ 2^3 \succ 2^2 \succ 2^1 \succ 2^0.$$

Sarkovskii's theorem affirms for a continuous map $f : \mathbf{R} \rightarrow \mathbf{R}$, which has a cycle of period P, to have cycles of periods that precede P in the above ordering.

In fact, if there is a cycle of period three, which has the highest rank in the ordering, then it is possible for the map to have infinitely many cycles simultaneously – but only one, at most, can be stable for an SN-unimodal map. Sarkovskii's result is also part of *the period three implies chaos theorem* of Li and Yorke [1975].[6] In addition, Li and Yorke showed that there exists an uncountable set $S \subset I$, called the *chaotic set* or *scrambled set*, containing only *aperiodic trajectories.* If the critical point is not attracted by a stable cycle, then the motion is confined to a trajectory that has no periodic points.

The time evolution of x_t, generated by the simple quadratic map, displays a remarkable transition from regular to chaotic behavior as the parameter a, which tunes the strength of the nonlinearity, is increased (see figure 1.1):

- For $a < 1$, the population decreases and eventually dies out.
- For $1 < a < 3$, the time path will converge to a stable fixed point equilibrium, which means that the population will eventually stay constant.
- For $a = 3.2$, the time path is trapped by a period-two cycle.
- For $a = 4$, the time path has infinitely many unstable periodic points but no stable periodic orbit i.e., *the motion is completely aperiodic.* Furthermore, the trajectory exhibits *sensitivity on initial conditions.* Such behavior is referred to as *deterministic chaos.*

The *bifurcation diagram*, which is also called the *Feigenbaum scenario*, in figure 1.2, summarizes the long run time evolution of x_t, for values of the parameter $2.9 \leq a \leq 4$. Such a diagram is constructed by iterating an

[6] This theorem is stated in its full length in the appendix of chapter two.

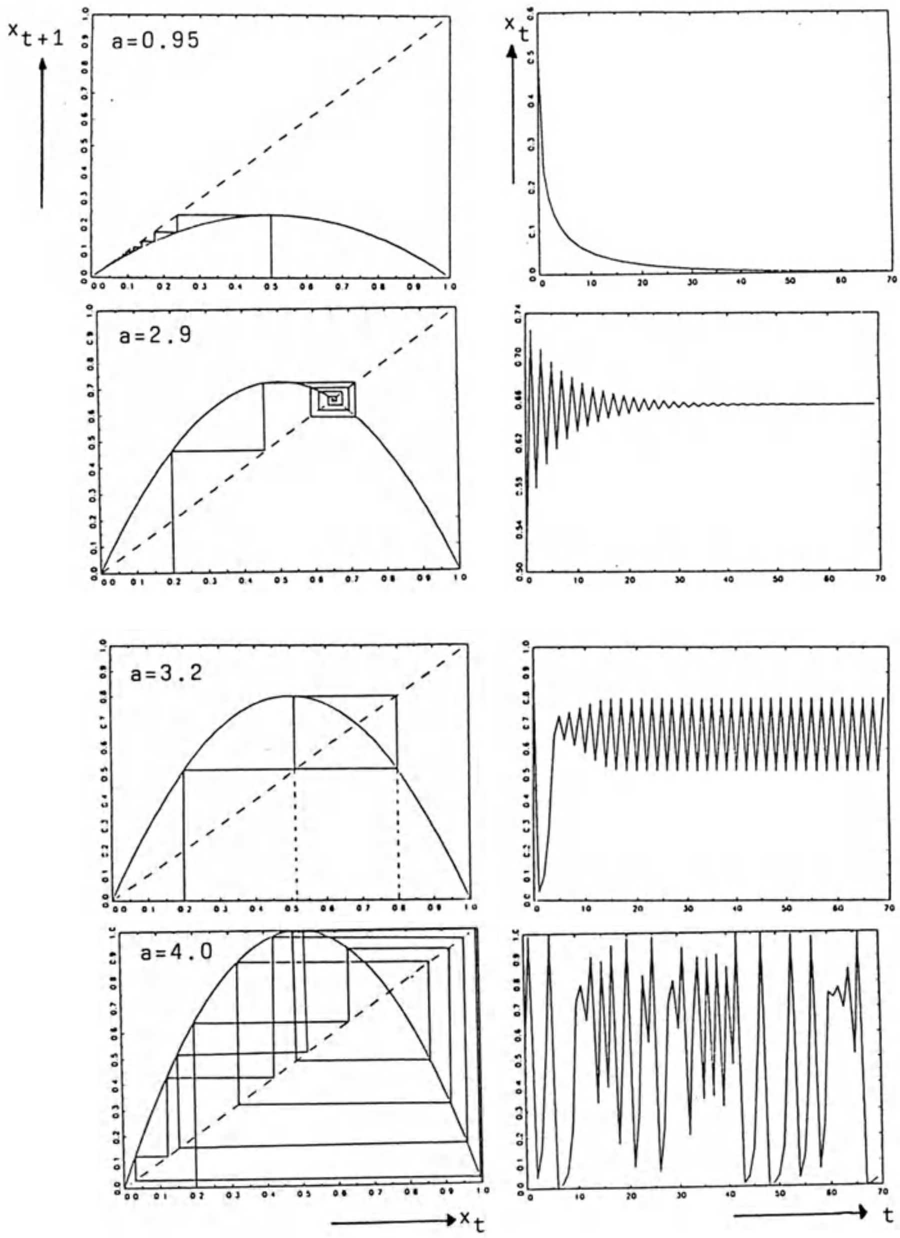

Figure 1.1: Phase curve of the logistic map for different values of the parameter a and a plot of the corresponding time path. The iterative process of some initial state x_0 if fully determined, and can be traced in the phase curve as follows: For x_0 find the value of the function $x_1 = f(x_0)$. Since x_1 is the starting value for the next iteration, it is projected back to the horizontal axis. Then, for x_1, find the value of the function $x_2 = f(x_1)$. This procedure is repeated for succeeding iterations.

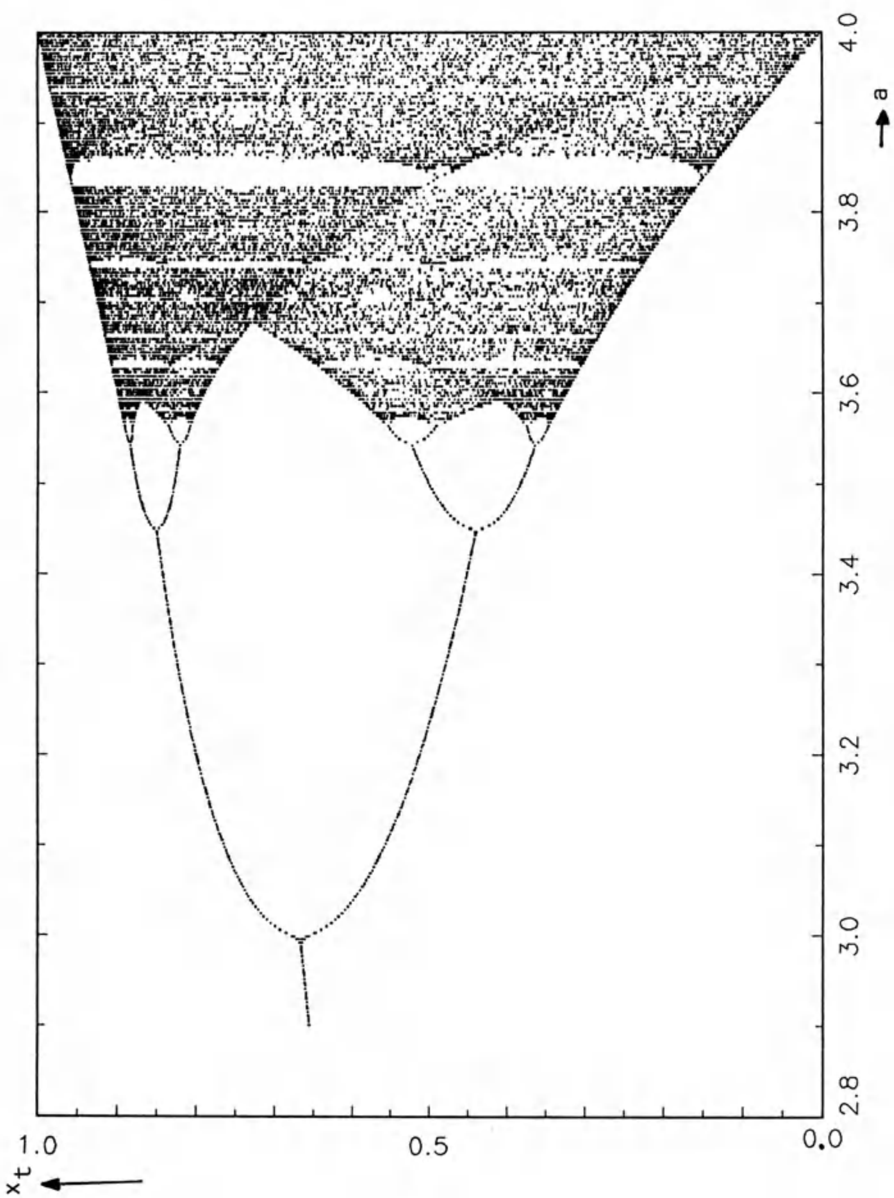

Figure 1.2: The bifurcation diagram is a multiple valued plot of the attracting set, as far as can be calculated with the aid of a computer, against the values of the parameter a.

initial state, say 500 times, and plot the last 400 iterations[7] against the value of the parameter a – thereby allowing the system a transient time of 100 periods. This procedure is repeated for increasing values of a. The dynamic behavior goes from a stable point, through a *sequence of period doubling bifurcations* into stable cycles of period 2,4,8,16,.... The parameter values a_i where a 2^i cycle is born converge to an *accumulation point* $a_\infty = 3.5699$ in a geometric progression. Then, a regime of "noisy periodicity" is entered where the motion is confined to a Cantor set that is not sensitive to initial conditions. The *strange attractor set* finally occurs for values of the parameter beyond the period three cycle which appears for $a = 3.8284$. This set is indicated by the shaded area in the bifurcation scenario, which is interrupted by small bands of periodic motion or *periodic windows.* Besides sensitivity to initial conditions, the dynamical behavior of this system may react vividly to miniscule changes in the control parameter: We may be in a chaotic regime but a small change in a may transform the system to cyclic behavior ("order and chaos are close to each other").

1.3 Higher dimensional systems

This section provides an idea of complex dynamics in higher dimensional systems. We seek a solution $[x(x_0, t)]$ to some dynamical system consisting of n equations, such as

$$\frac{dx(t)}{dt} = F(x(t)) \quad x \in \mathbf{R}^n \tag{1.4}$$

in the continuous case, and

$$x(t+1) = F(x(t)) \quad x \in \mathbf{R}^n \tag{1.5}$$

in the discrete case, where $F : \mathbf{R}^n \to \mathbf{R}^n$. In an n-dimensional dissipative[8] system the orbit $x(t)$ contracts space volume in the course of time, which means that the motion is confined to a space of dimension smaller than n.

In a continuous system, a one-dimensional flow must settle down on a limit point of zerǒ dimension, whereas a two-dimensional flow can also settle down

[7]The number of iterations that can be plotted often depends on the capacity and precision of the computer program. In general, the more iterations that can be plotted, the more structure is visible.

[8]From the physical point of view, dissipative systems are characterized by a permanent consumption of energy which dissipates through the system. From the point of view of information theory, dissipation implies the consumption of information on the transient, and hence loss of memory of the initial condition. If the motion is confined to a strange attractor set, then information is permanently consumed by the system.

on a limit cycle i.e., the attractor is a closed loop in state space. Chaotic motion means an irregular wandering of the trajectory within a bounded region of the state space, where it seems to display a certain recurrence since the trajectories from different initial conditions can come arbitrarily close to one another. Yet, each oscillation is unique, no pattern is ever exactly repeated, and the true period is infinite. Besides being aperiodic, the motion on the chaotic attractor is *sensitive to initial conditions*: Near neighboring points locally diverge exponentially in the course of time, but eventually, all trajectories are caught by the bounded attractor set. This phenomenom is remarkable indeed and attractors that display this kind of behavior are called *strange attractors.*

Obviously, since continuous trajectories (or flows) in general cannot intercept each other on a chaotic attractor, it must be the case that *chaotic motion is only possible in three- or higher-dimensional continuous systems.*[9] This is not valid for discrete time systems (or equivalently *point mappings, maps*, or simply *iterations*): Since time t is a sequence of integers, trajectories starting from any initial condition can go everywhere in state space without ever sharing a common point. As a consequence, aperiodic behavior can already occur in discrete systems of dimension one, such as the logistic map, and of dimension two, such as the Hénon map. In the sequel, we will give only an idea of some popular nonlinear systems, mainly by plotting their strange attractors as far as we can compute them. For introductory purposes, these plots will give a good idea of how an irregular wandering of a trajectory can look like.

One of the most celebrated chaotic continuous dynamical systems is the *Lorenz attractor.* At the beginning of the 1960s the meteorologist E. Lorenz found that his simplified system of three coupled differential equations

$$\begin{aligned} \dot{x} &= \sigma x + \sigma y \\ \dot{y} &= -yx + rx - y \\ \dot{z} &= xy - bz \end{aligned} \tag{1.6}$$

exhibits sensitive dependence on initial conditions for wide ranges of the parameters σ, r, b (one often takes $\sigma = 10, r = 28, b = \frac{8}{3}$). Figure 1.3 shows a time plot and a state space[10] trajectory of this complicated attractor spiralling around, and jumping between, two wings.

In relation to the Lorenz attractor, Rössler [1976] introduced a similar strange attractor with only one spiral arising from a simpler three-dimensional system

[9] It follows from the Poincaré-Bendixson theorem (also stated in Lorenz [1989]) that no attractor more complex than a limit cycle can occur in a two-dimensional system.

[10] The space that is spanned by the axes of the variables that are dynamically connected is referred to as state space.

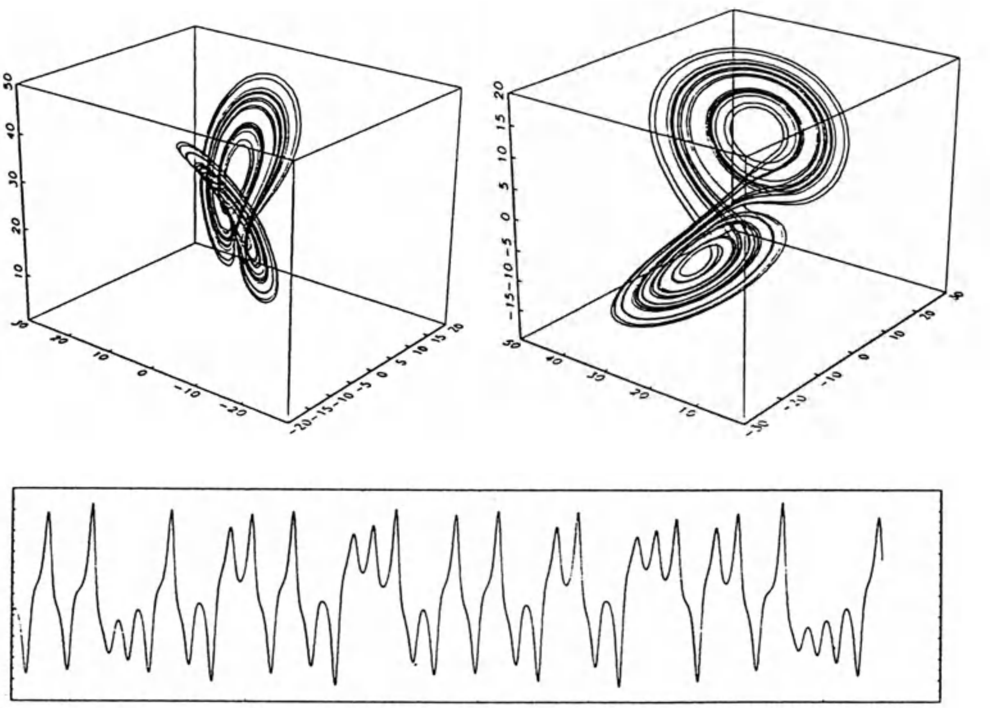

Figure 1.3: A trajectory and time plot of the x coordinate of a solution to the Lorenz model. The model itself is a simplified version of a fluid layer heated from below and cooled from above, representing the atmosphere heated from below by the earth's absorption of sunlight and losing heat into space. The variable x describes the convective motion, y stands for the horizontal temperature variation, and z represents the vertical temperature variation. The parameters σ, r, and b are proportional to physical constants (the Prandl number, the Rayleigh number, and the size of the region under scrutiny, respectively).